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ON THE TABLES OF PLANETARY VISIBILITY IN THE ALMAGEST AND THE HANDY TABLES

BY

ASGER AABOE

København 1960
i kommission hos Ejnar Munksgaard

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I.

Of Ptolemy's Handy Tables, the detailed astronomical tables which were the outcome of the theories developed in his Almagest,[1] the preface[2] alone has survived. The tables themselves, which played an important rôle in Islamic and early Western astronomy, have reached us only in the edition of Theon of Alexandria.[3] We are thus faced with the problem of deciding how faithfully these Theonic tables do represent the Ptolemaic originals, and this problem is, indeed, a very real one, for it is obvious that the Handy Tables, as we have them, have not in their entirety been directly derived from the Almagest: there are, in places, discrepancies both in parameters and in methods.

The only manner in which we can resolve this problem with the textual material now at hand is either by demonstrating agreement between Ptolemy's preface and Theon's tables, particularly of course at points where the likelihood of such agreement a priori is slight, or alternatively by showing disagreement between the two texts. Except in unimportant matters, such as the arrangement of the tables, no such disagreement has turned up. Furthermore, there is perfect accord between Ptolemy's prefatory instructions and the Theonic tables in one of the instances where the latter deviate, and substantially so, from the Almagest's tables and theories, viz. on the question of planetary latitudes, as can be seen from a paper of van der Waerden.[4] And, as will appear, the follow-

[1] Text edited by J. L. Heiberg, Leipzig, 1898, 1903; German translation by K. Manitius, Leipzig, 1912, 1913.

[2] *Ptol. Opera II*, ed. Heiberg, Leipzig, 1907, pp. 159—185.

[3] Theon flourished during the latter half of the fourth century A. D. (thus some 200 years after Ptolemy). The only printed edition of the Handy Tables is in Halma, *Commentaires de Théon etc.*, Paris, 3 vols., 1822—25. The visibility tables discussed in this paper are in vol. 3, pp. 16—31. A new edition of the Handy Tables, by W. D. Stahlman, will appear soon.

[4] B. L. van der Waerden: *Bemerkungen zu den Handlichen Tafeln des Ptolemaios*, Sitzungsberichte der Bayerischen Akademie der Wissenschaften, Math.-nat. Klasse, 1953, Nr. 23 (pp. 261—272).

ing analysis of the other major point of disagreement between the Almagest and the Handy Tables, viz. on the matter of planetary visibility, bears out in a most surprising fashion the corresponding remarks in Ptolemy's preface. Thus it seems beyond reasonable doubt that the Handy Tables, as they are known to us, are indeed the Ptolemaic originals, save for small editorial changes and unimportant additions such as special tables for Byzantium.

The primary aim of the following investigation[1] is, however, to lay bare the internal structure of a pair of corresponding sections of the Almagest and the Handy Tables. These sections are concerned, as hinted above, with the problem of first and last visibility of the planets or, to use the Greek terminology, their phases: when, say, an outer planet is near conjunction with the sun it is invisible; but as the sun leaves it farther and farther behind there will come a morning when the planet rises in sufficient darkness for it to be visible, if only for a short while, before sunrise. This is the moment of first visibility. The phenomenon of last visibility happens in a symmetrical fashion.

I shall, following Neugebauer, use the following notations:

$$\left.\begin{array}{ll} \mathit{\Gamma}\text{:} & \text{first appearance} \\ \mathit{\Omega}\text{:} & \text{last appearance} \end{array}\right\} \text{ of an outer planet.}$$

$$\left.\begin{array}{ll} \mathit{\Gamma}\text{:} & \text{first appearance as a morning star} \\ \mathit{\Sigma}\text{:} & \text{last appearance as a morning star} \\ \mathit{\Xi}\text{:} & \text{first appearance as an evening star} \\ \mathit{\Omega}\text{:} & \text{last appearance as an evening star} \end{array}\right\} \text{ of an inner planet.}$$

II. The Almagest

These phenomena are treated by Ptolemy in Almagest XIII, 7—10. The criterion for first or last visibility is that a certain critical altitude difference between the planet and the sun has been reached at sunrise ($\mathit{\Gamma}$ and $\mathit{\Sigma}$) or sunset ($\mathit{\Xi}$ and $\mathit{\Omega}$). Ptolemy's fundamental assumption is that this critical altitude difference—the *arcus visionis*—depends on nothing save the planet. He selects one phenomenon for each planet taking place at the beginning of Cancer, because of the favourable visibility conditions at mid-

[1] Part of the work was done while I was the recipient of a Tufts University Faculty Research Fellowship for which I wish to express my gratitude.

summer. As for terrestrial latitude Ptolemy chooses that of Phoenicia (longest daylight $14\frac{1}{4}^h$) because "most of the reliable observations have been made by the Chaldeans at this latitude, and in Greece and Egypt on either side thereof" (XIII, 7)—a reference to the central rôle which these phenomena played in Babylonian astronomy[1]. Finding the elongation, E, and the latitude, β, of the planets, presumably from the dates (unfortunately he omitted

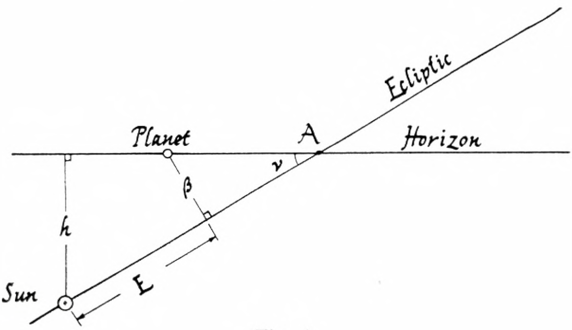

Fig. 1.

the selected observations themselves and most details of how E and β were derived), he computes the arcus visionis, h, for each, treating the problem as one in plane rather than spherical trigonometry (v. fig. 1) from his equivalent of

$$h = E \sin \nu + \beta \cos \nu \tag{1}$$

where ν is the angle between the ecliptic and the horizon, and where β is to be counted with sign in the usual fashion. He obtains the following values of h:

for Saturn:	11°	Venus:	5°
Jupiter:	10°	Mercury:	10°
Mars:	$11^1/_2{}^\circ$		

The variation in h reflects, of course, the difference in brightness between the planets: the brighter the planet, the smaller its arcus visionis.

Ptolemy now reverses this process, and finding β in a manner which will be discussed below, computes from (1) the necessary elongation of each planet for first and last appearance, for the

[1] v. O. Neugebauer: *Astronomical Cuneiform Texts*, I—III, London, 1955.

beginning of each zodiacal sign, all for the terrestrial latitude of Phoenicia (Ptolemy's value appears to be slightly more than 33°). The results are gathered in a table in Almagest XIII, 10. From a day by day ephemeris giving the longitude of a certain planet and of the sun one can then, for the latitude of Phoenicia, determine when the planet's first or last appearance will occur, namely when the critical elongation corresponding to its longitude, and given in the table, is attained.

Ptolemy treats of this entire problem in a somewhat summary fashion which is quite different from his usual explicit, painstaking, and detailed manner of presentation. It is, however, clear from the examples in Almagest XIII that it is the point of intersection between the horizon and the ecliptic (A in fig. 1) which assumes the precise longitudes 0°, 30°, 60°, . . ., and therefore the determination of v is a simple problem in spherical trigonometry of the type discussed in Almagest II, 11. Thus the main point which Ptolemy leaves unexplained is how β was determined. Three parameters have to be known if one is to find the latitude of a planet according to the Ptolemaic theory[1]: (i) the longitude of the apogee of the deferent, (ii) the mean longitude of the planet which, with (i), yields the position of the centre of the epicycle on the deferent, and (iii) the anomaly, i.e. the position of the planet on the epicycle. The only change to which (i) is subject is the precession; Ptolemy's value is 1° per century, so its effect can be ignored for quite a long span of years. (ii) and (iii) could immediately be found from the proper tables in the Almagest if the date of the phenomenon were known. But here the situation is different. However, if the longitude of A, and so v, is given, it is simple to determine (v. fig. 1) the longitude of the sun, and so of the mean sun, for h is fixed. This determines in essence one of the two remaining parameters since, for an inner planet, the mean longitude is that of the mean sun while, for an outer planet, the radius of the epicycle is parallel to the direction to the mean sun. It appears, again from his examples, that Ptolemy probably took this step in each case for the inner planets. It is, however, far from clear how he proceeded to find the other parameter, and so β. In

[1] For a summary of Ptolemy's planetary models, as far as motion in longitude is concerned, see O. Neugebauer: *The Exact Sciences in Antiquity*, 2nd ed., Providence, 1957, appendix I.

Celestial Latitude, β, of the Planets & Angle, ν, between Horizon & Ecliptic for Phoenicia, extracted from Almagest XIII, 10.

	β of ☿				β of ♀				β of ♃	β of ♄	ν	
0° of	Ξ	Ω	Γ	Σ	Ξ	Ω	Γ	Σ	Γ & Ω	Γ & Ω	East	West
♈	1;0	2;36	-3;40	-3;26	-0;33	5;40	4;0	-0;50	-1;10	-2;0	33;0	80;50
♉	0;50	0;4	-3;10	-3;10	0	3;35	1;46	-0;34	-1;0	-1;45	34;30	77;0
♊	1;22	-2;4	-2;30	-1;23	0;34	0;44	-1;29	-0;1	-0;35	-1;0	41;10	66;30
♋	1;40	-3;30	-1;37	0;20	1;0	-2;30	-4;20	0;30	0	0	51;30	51;30
♌	1;15	-4;13	-1;14	1;15	1;9	-4;49	-6;0	0;50	0;35	1;0	66;30	41;10
♍	-0;15	-3;50	-0;22	1;14	1;0	-6;20	-6;20	1;0	1;0	1;45	77;0	34;30
♎	-2;54	-3;10	1;45	0;48	0;55	-5;32	-3;50	1;2	1;10	2;0	80;50	33;0
♏	-3;0	-3;0	2;20	-0;12	0;21	-3;24	-1;25	0;58	1;0	1;45	77;0	34;30
♐	-2;40	-2;0	3;25	-1;0	-0;13	-0;23	1;50	0;25	0;35	1;0	66;30	41;10
♑	-1;25	0;22	4;0	-1;38	-0;37	2;50	4;36	-0;15	0	0	51;30	51;30
♒	-0;40	2;28	2;22	-2;20	-0;56	5;2	6;13	-0;52	-0;35	-1;0	41;10	66;30
♓	0;22	2;24	-1;6	-2;50	-1;0	6;30	6;30	-1;0	-1;0	-1;45	34;30	77;0

Table 1.

order to throw light on this problem I found the values of β which he actually employed when computing the tables. From the examples, and by the aid of certain regularities of the tables which give E, the values of ν used by Ptolemy were restored; it is to be noted that the tables in Almagest I, 13 are of no avail since they are concerned with the angles between the ecliptic and altitude circles for the seven climates only, and Phoenicia lies about half-way between the third and the fourth[1]. These values of ν are given in the last two columns of Table 1[2]. Formula (1) then yields the values of β which are listed in the first columns of Table 1, and which are represented graphically in figs. 2, 3, and 4. The values of β for Mars are omitted; all that I can say with safety is that they are small but not identical to 0.

We see that in the cases of both Jupiter and Saturn the following simple features appear: first, for a given zodiacal sign the β's for Γ and Ω are equal; second, the β-curves are pure sine waves with amplitudes that are rounded-off values of the maximal lati-

[1] Ptolemy's seven climates are the parallels characterized by duration of longest daylight of 13^h, $13\frac{1}{2}^h$, 14^h, ..., 16^h; as mentioned, Phoenicia has longest daylight of $14\frac{1}{4}^h$.

[2] Following the accepted practice for transcribing Babylonian sexagesimal fractions, I write 80°;50 for $80\frac{50}{60}°$ (or 80°50′), 80°;50,30 for $80° + \frac{50}{60}° + \frac{30}{60^2}°$ (or 80°50′30″), etc.

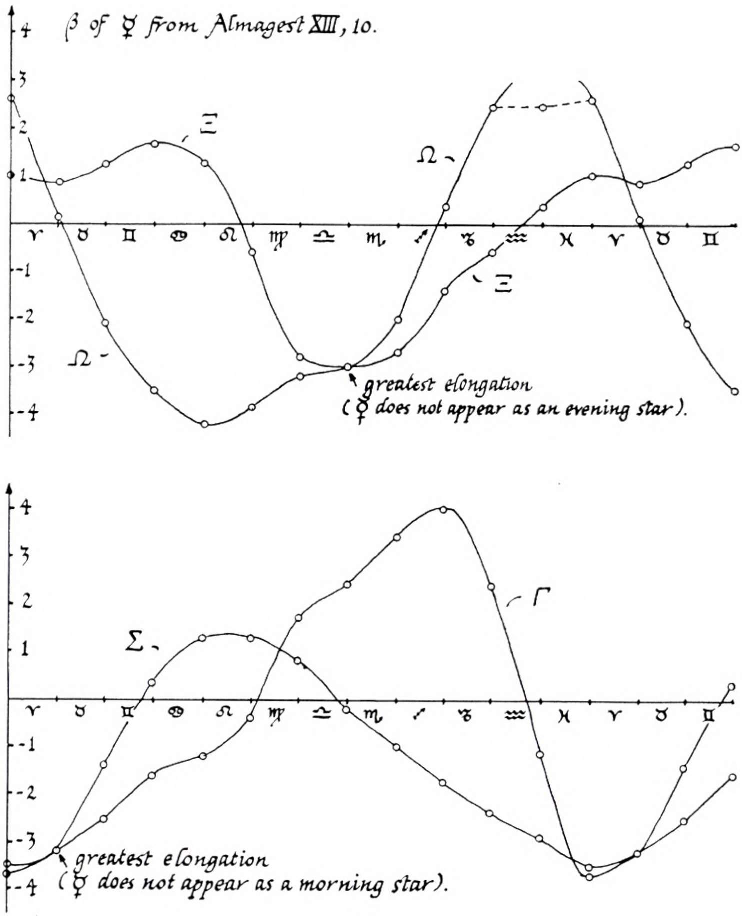

Fig. 2.

tude when the planet is in conjunction; and third, the ascending nodes of both planets are placed at ♋ 0°. Since the last point involves but a slight adjustment of the Almagest parameters, these simplifications mean that for the purpose of finding the latitude of Jupiter and Saturn, Γ and Ω are identified with conjunction, and that the effects (here slight) of the eccentricity of the deferent have been ignored. These labour-saving measures are quite justified by the smallness of the quantities involved.

But where Venus and Mercury are concerned the situation is different. Each phenomenon has its distinct β-curve. As far as I can

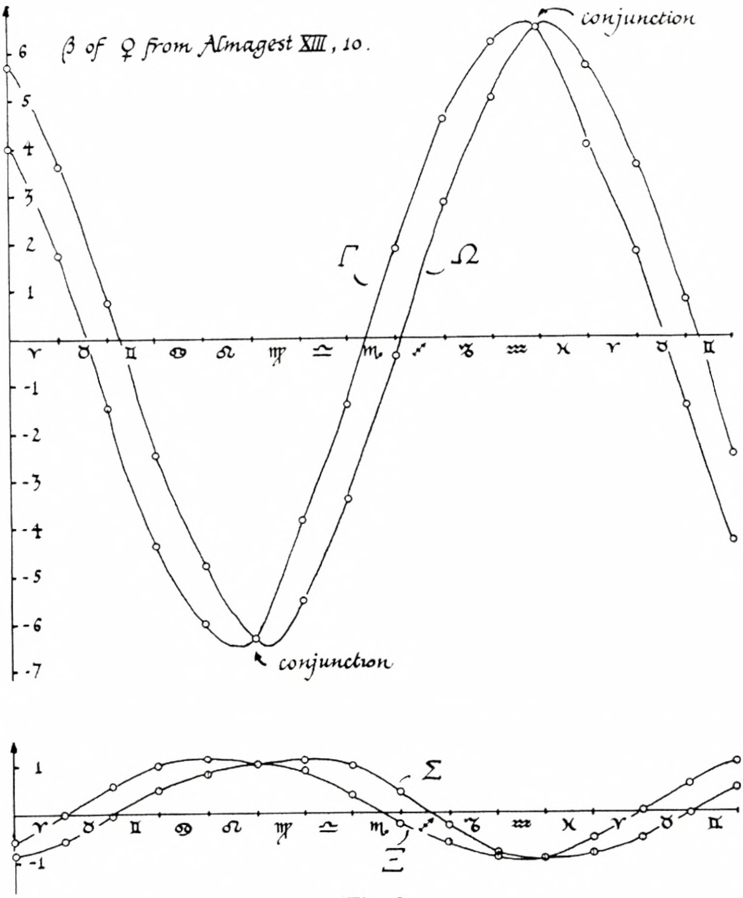

Fig. 3.

see, Ptolemy did not, when computing these latitudes, use any simple unifying hypothesis such as, what might have been natural in view of later texts[1], the assumption of a fixed position of the planet on the epicycle corresponding to each phenomenon. This assumption, in particular, can be discounted because, as is exhibited on the respective figures, there are special instances where Venus, at Γ and Ω, is supposed to be at the perigee of its epicycle, and where Mercury, at Σ and Γ, and at Ξ and Ω, is assumed to be

[1] From Professor Neugebauer I have the following parameters which he extracted from *Catalogus Codicum Astrologorum Graecum* (Bruxelles, 1898—1953),

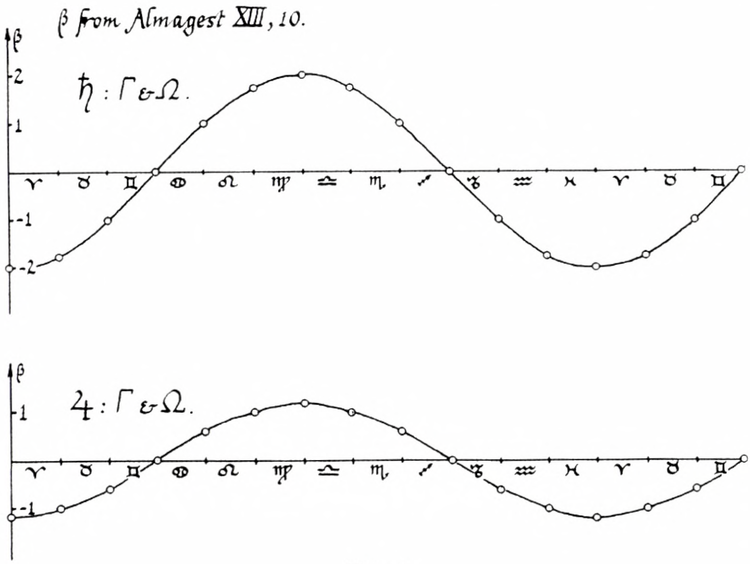

Fig. 4.

at its maximal elongation from the sun. These special cases are found in the examples in Almagest XIII, 8, where Ptolemy shows that his theory is capable of explaining that Venus changes from an evening to a morning star in at most two days when at the beginning of Pisces, while it uses 16 days at the beginning of Virgo, and that Mercury does not appear at all as an evening star at the beginning of Scorpio, nor as a morning star at the beginning of Taurus, because its elongation is insufficient when these phenomena are due in the sequence of synodic events.

A safe reconstruction of the various anomalies which were assigned to Venus and Mercury is, I fear, a hopeless task in view of the complexity of the latitude theory in the Almagest and the relatively small variation of the latitude with the anomaly. My belief is, nonetheless, that a fresh decision was made in each

vol. 7, p. 119ff., and from Vat. Gr. 208, fol. 131 r. α denotes the position on the epicycle (counted from its apogee) of the various planets at first and last appearance:

$$\text{Saturn:}\begin{cases}\Gamma\colon \alpha = 17^\circ\\ \Omega\colon \alpha = 343^\circ\end{cases}\qquad \text{Venus:}\begin{cases}\Xi\colon \alpha = 12^\circ;\ 24\\ \Sigma\colon \alpha = 347^\circ;\ 36\end{cases}$$

$$\text{Jupiter:}\begin{cases}\Gamma\colon \alpha = 16^\circ\\ \Omega\colon \alpha = 344^\circ\end{cases}\qquad \text{Mercury:}\begin{cases}\Xi\colon \alpha = 38^\circ\\ \Sigma\colon \alpha = 322^\circ\end{cases}$$

$$\text{Mars:}\begin{cases}\Gamma\colon \alpha = 42^\circ\\ \Omega\colon \alpha = 318^\circ\end{cases}$$

instance, perhaps as the result of an iterative process; and this view seems to be corroborated by the al-Khāzinī tables which I shall discuss below in section V.

III. The Handy Tables

The section of the Handy Tables concerning planetary visibility consists of a table for each phenomenon for the several planets. These list the critical elongations for the beginning of each zodiacal sign and for each of the seven climates in the fashion shown in Table 2. If they were simply extensions of the visibility

Phases of Evening Setting of Saturn from 181° to 360°.							
Signs	I	II	III	IV	V	VI	VII
	13^h	$13\frac{1}{2}^h$	14^h	$14\frac{1}{2}^h$	15^h	$15\frac{1}{2}^h$	16^h
Aries	13;21	13;0	13;31	13;46	14;10	14;42	15;15
Taurus	13;50	13;6	13;33	14;7	14;40	15;3	15;54
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Table 2.

table in the Almagest, then the entries in the latter should fit between the corresponding columns for the third (Lower Egypt) and the fourth climate (Rhodes); for these climates are characterised by durations of longest daylight of 14^h and $14\frac{1}{2}^h$, respectively, and Phoenicia's longest daylight is, as noted, $14\frac{1}{4}^h$. This, however, is far from being the case. Furthermore one can see immediately that if the Handy Tables also use the assumption of a constant arcus visionis for each planet, then these arcs differ considerably from those in the Almagest. Indeed, the following values can be directly read off from the tables, as van der Waerden[1] has remarked:

for Saturn:	$13°$	Venus (Γ and Ω):	$5°$
Jupiter:	$9°$	Venus (Σ and Ξ):	$7°$
Mars:	$14\frac{1}{2}°$	Mercury:	$12°$

for in the second climate (Syene) the angle ν between the horizon and the ecliptic is $90°$ at Libra $0°$ in the East, and at Aries $0°$ in

[1] l.c. in note 4, p. 3

the West, and hence, from (1), $E = h$ regardless of what value β may have (see again Table 2).

We shall now proceed to analyse the visibility tables in detail:

1°. The two tables for Mars are conspicuous for two reasons: first, that each table is symmetrical in the sense that it holds for any climate that

$$E\,(\lambda) = E\,(360^\circ - \lambda)$$

so that, e.g., we find the same entries for Taurus as for Pisces; second, that the table for Γ is identical with the table for Ω rotated

Angle, ν, between Horizon & Ecliptic, computed from Alm. I,13.

Climate:	I	II	III	IV	V	VI	VII	
Ter. Lat.:	16;27	23;51	30;22	36;0	40;56	45;1	48;32	
0° of: ♈	49;42	42;18	35;47	30;9	25;13	21;8	17;37	♎
♉ ♓	52;9	44;41	37;55	32;7	26;59	22;43	19;22	♏ ♍
♊ ♒	59;51	51;53	44;49	38;36	33;6	28;25	24;20	♐ ♌
♋ ♑	71;57	63;46	56;28	50;1	44;16	39;21	34;58	♑ ♋
♌ ♐	84;51	76;53	69;49	63;36	58;6	53;25	49;20	♒ ♊
♍ ♏	94;9	86;41	79;55	74;7	68;59	64;43	61;22	♓ ♉
♎	97;24	90;0	83;29	77;51	72;55	68;50	65;19	♈
East								West

Table 3.

180° so that, e.g., the entries for Γ in Taurus are the same as those for Ω in Scorpio. It is therefore a reasonable assumption that if the tables are computed on the basis of (1), β of Mars must be identical to 0°, or that the entries simply are

$$E = \frac{h}{\sin\nu}$$

where $h = 14\frac{1}{2}°$.

Values of ν can thus be computed, sign by sign and climate by climate, and it appears that they agree, within the error of computation, with those derived from the tables in Almagest I, 13 (v. Table 3).

2°. We can now, using these values of ν and the above values of h, find β corresponding to each E in the remaining tables, assuming once more that they are computed according to (1).

This assumption is amply confirmed, for the β's found in this fashion remain constant throughout each zodiacal sign, i.e. β is independent of climate.

Furthermore the following regularities appear:

3°. For a given planet, a zodiacal sign yields the same β for Γ and Ω, and for Σ and Ξ. That this is so is corroborated by the

Celestial Latitude, β, of the Planets, extracted from the Visibility Tables in the Handy Tables.

	♄	♃	♂	♀		☿	
	$\Gamma\,\Omega$	$\Gamma\,\Omega$	$\Gamma\,\Omega$	$\Gamma\,\Omega$	$\Sigma\,\Xi$	$\Gamma\,\Omega$	$\Sigma\,\Xi$
♈	-2;0	-1;0	0	7;10	-1;10	0;45	~ 0
♉	-1;50	-0;50	0	3;45	-0;30	-1;10	0;50
♊	-1;20	-0;30	0	-0;40	0;20	-2;45	1;30
♋	-0;20	0	0	-5;0	1;0	-3;45	1;45
♌	0;30	0;30	0	-8;0	1;30	-3;40	1;40
♍	1;30	0;50	0	-8;40	1;30	-2;40	1;15
♎	2;0	1;0	0	-7;10	1;10	-0;45	~ 0
♏	1;50	0;50	0	-3;45	0;30	1;10	-0;50
♐	1;20	0;30	0	0;40	-0;20	2;45	-1;30
♑	0;20	0	0	5;0	-1;0	3;45	-1;45
♒	-0;30	-0;30	0	8;0	-1;30	3;40	-1;40
♓	-1;30	-0;50	0	8;40	-1;30	2;40	-1;15

Table 4.

following fact which is independent of my computations: for a given planet, the lines corresponding to Cancer are identical in the tables for Γ and for Ω, and also in those for Σ and for Ξ. Since ν is the same for Cancer 0° in East and West this means that β is the same for the pairs of phenomena. The situation is the same for Capricorn.

4°. In each table β corresponding to a certain zodiacal sign has the opposite sign but the same numerical value as the β corresponding to the line six signs later, i.e.

$$\beta(\lambda) = -\beta(\lambda + 180°).$$

Table 4 lists the values of β extracted from the tables, and they are graphically represented in figs. 5 and 6.

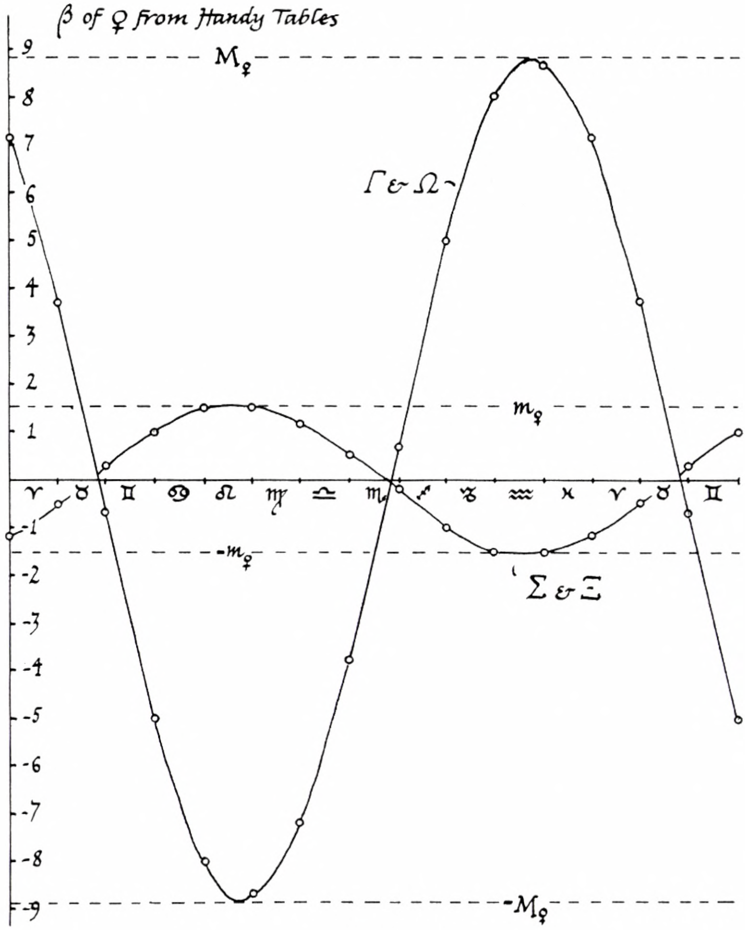

Fig. 5.

The horizontal dotted lines ($\beta = \pm M$ or $\pm m$) in these graphs represent the following values:

Mercury:	$M = 3°; 52$
	$m = 1°; 46$
Venus:	$M = 8°; 51$
	$m = 1°; 28$
Jupiter:	$M = 2°; 03$
Saturn:	$M = 1°; 05.$

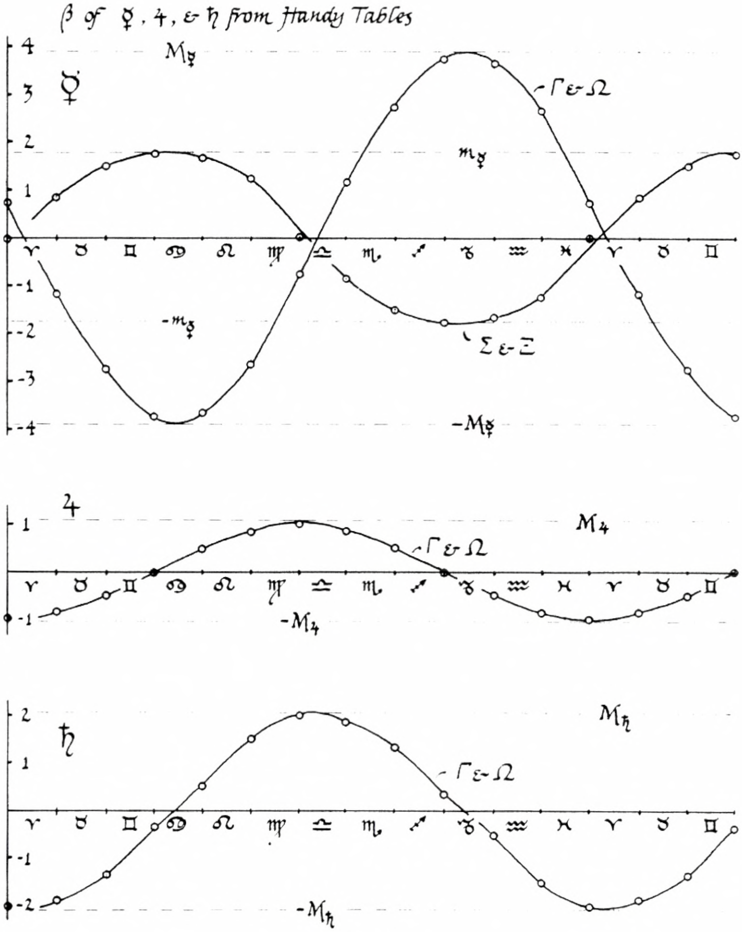

Fig. 6.

These parameters come from the Handy Tables. For an inner planet M and m are the maximum and minimum values, respectively, of the column C (Γ in mss.) of the latitude tables, and so they represent the maximal latitude of the planet at inferior and superior conjunction, respectively, (i.e. anomaly $= 180°$ and $0°$) when the centre of the epicycle is at mean distance from the Earth. For an outer planet M is the maximal latitude of the planet at conjunction (anomaly $= 0°$) when the centre of the epicycle is at mean distance, for M is found as the difference between i and the minimum value in column C (Γ in mss.) in the respective latitude tables where for

$$\begin{array}{ll} \text{Jupiter:} & i = 1\tfrac{1}{2}^\circ \\ \text{Saturn:} & i = 2\tfrac{1}{2}^\circ, \end{array}$$

denoting the inclination of the deferent against the ecliptic. For Mars the latitude table yields:

$$M = 1^\circ - 0^\circ;54 = 0^\circ;6.$$

The β curves in the figures are pure sine waves with these M and m as amplitudes.

To summarize:

The tables of planetary visibility in the Handy Tables are computed from

$$E = \frac{h}{\sin \nu} - \beta \cot \nu$$

where h *is fixed for each planet save Venus, to which is assigned one value for* Γ *and* Ω *and another for* Σ *and* Ξ; *and where the* ν *are identical with those found from Almagest I, 13. For the purpose of finding the latitude,* β, *from the proper tables in the Handy Tables, the phenomena are identified with the nearest conjunction (anomaly* 0° *or* 180°*), these conjunctions are assigned the longitudes* 0°, 30°, . . ., *and the eccentricity of the deferent is ignored. For Mars the latitude is, however, set equal to* 0° *throughout, a natural consequence of* M *being but* $0^\circ; 6$.

The visibility tables are thus completely explained.

IV.

It is now evident that the Handy Tables, while maintaining the basic assumption of a constant arcus visionis for each planet (with the exception of Venus), depart from the Almagest in two directions, viz. as to the values of these arcs, and in the manner in which the latitudes of the planets are computed.

The new arcus visionis values are generally more conservative, and probably better, than their Almagest counterparts, and the distinction between Venus near superior and near inferior conjunction is a much needed amendment to the Almagest which, in this connection, ignores the extreme variation in brightness of Venus.

The other departure from the Almagest's techniques is, however, surely motivated by a desire for expediency rather than for greater exactitude, for here the simplifying measures of identifying first and last appearance with conjunction and of ignoring eccentricity, which in the Almagest were taken, justifiably, in the cases of Saturn and Jupiter, are applied also to Venus and Mercury where no such justification exists. It is a quite complicated matter to ascertain the final influence of these simplifications on the dates of first and last appearance which, of course, are the ultimate aims of the entire theory. Suffice it to say here that the daily change in elongation of Venus near inferior conjunction is of the order of $1\frac{2}{3}°$, the size of which tends to counteract somewhat the quite large errors in latitude induced by the simplifications; on the other hand, the daily change in elongation of Mercury during its visibility is quite small, Mercury being near its greatest elongation, and this will have as a result that an inaccuracy in the latitude will appear greatly magnified in its effect on the date.

One would not expect Ptolemy to abandon his refined latitude theory for cruder methods in one of the few problems where the latitude is of decisive importance. It was therefore greatly surprising to find that precisely this point furnishes the evidence for Ptolemy's authorship of the tables; for in the section on the phases of the planets in his preserved preface to the Handy Tables Ptolemy says:

For the correction due to latitude we assumed the one which arises in the apogee and the perigee of the epicycles[1]

which is exactly what we found.

This strong, if not, indeed, conclusive evidence is corroborated by a fact which by itself would not carry much weight, viz. that the ascending nodes of the planets, which are subject to precession, have longitudes (see figs 5 and 6) which, with the Handy Table parameters, fit Ptolemy's time well.

Thus, as stated in the introductory remarks, Ptolemy's authorship of the Handy Tables appears to be beyond reasonable doubt.

The reason for Ptolemy's crude latitude computations in the visibility section of the Handy Tables, as well as for his summary

[1] ᾗ πεποιήμεθα τοῦ πλάτους διορθώσει τῇ περὶ τὰ ἀπόγεια καὶ τὰ περίγεια τῶν ἐπικύκλων συνισταμένῃ (HEIBERG, *Ptol. Opera II*, p. 174, No. 15).

treatment of this question in the Almagest, may be that this topic no longer was of great interest to him and that he included it at all only out of deference for the central position which it used to hold in mathematical astronomy.

V. Visibility Tables in the Islamic Zījes

With the visibility tables in the Handy Tables are explained several corresponding sections in Muslim zījes. Thus the zīj of al-Battānī contains a table for planetary visibility which is identical with what the Handy Tables give for the fourth climate (Rhodes), and so the question which NALLINO[1] left open in his edition has been completely answered.

While the present paper was in preparation I learnt of a paper by E. S. KENNEDY and MUHAMMAD AGHA[2], of the American University of Beirut, dealing with planetary visibility tables in the known zījes (seven contain such tables), and a manuscript of this paper was placed at my disposal. It appears from this that the Sanjarī Zīj of ʿAbd al-Raḥmān al-Khāzinī[3] (c. 1120) contains a set of visibility tables for all seven climates, as those in the Handy Tables, but using the Almagest arcus visionis values. Professor KENNEDY sent me transcriptions of some of these, and I investigated in particular the tables for Venus at Γ and Ω (i.e. near inferior conjunction) for latitude, these cases being most likely to show clear trends. I shall refrain from giving any of the voluminous numerical material but content myself by citing the following results which I consider secure:

1°. The latitude of Venus at Γ or at Ω does not remain constant for a given zodiacal sign but varies with the elongation through

[1] On the problem concerning the latitudes used in the computation of these tables NALLINO says: *Quanam ratione latitudines planetarum in tabulis his conficiendis supputaverint, nec Ptolemaeus nec al-Battānī docent; frustra ego permultum temporis et laboris in tabularum constructionem enucleandam impendi. Nec rem attentaverunt (aut fortasse attentatam dereliquerunt) Purbachius, Regiomontanus, Maginus, Riccioli aliique ab XV at XVII saec. astronomi, et Delambre atque recentiores qui historiam astronomiae narraverunt. Impossibile igitur fuit, numeros codicis certissime emendare.* (NALLINO, *Al-Battani sive Albatenii Opus Astronomicum*, 3 vols., Milan, 1899—1907, vol. II, p. 258).

[2] E. S. KENNEDY and MUHAMMAD AGHA, *Planetary Visibility Tables in Islamic Astronomy*, to appear in Centaurus. My thanks are due to Professor KENNEDY for giving me free hands with the manuscript, as well as for sending me transcriptions of sections of the Sanjarī Zīj.

[3] No. 27 in E. S. KENNEDY, *A Survey of Islamic Astronomical Tables*, Trans. Am. Phil. Soc., 1956, Vol. 46, Part 2, pp. 123—177.

the seven climates in a fashion consistent with the latitude theory (Mercury seems to behave in a similar manner). Thus it is perfectly clear that a new value of the anomaly was chosen for each individual situation, which bears out my feeling about the procedure behind the Almagest tables.

2°. The latitudes of Venus which were used for Γ and Ω in the Almagest (see Table 1) do not consistently fit between the corresponding latitudes for the third and the fourth climate used in the al-Khāzinī tables. To be sure, the deviations are only rarely as much as 2°, yet they are large enough to make it clear that the tables were not computed simultaneously.

Thus we see that despite the differences mentioned in 2°, the al-Khāzinī tables are indeed in the Almagest tradition not only because they employ the same values for arcus visionis, but also in the sense of drawing full advantage of the refinements of the latitude theory.

All that can be said about the date of the computation of the al-Khāzinī visibility tables is, according to Kennedy and Agha, that it belongs to the interval bounded by Ptolemy and al-Khāzinī (the endpoints included), which is not very satisfactory, but one can still hope that a closer investigation of these important tables may yield a clew to their author.

The planetary visibility tables in the other six zījes belong to one or two of the three types discussed above; for details the reader should consult the forthcoming paper by Kennedy and Agha as well as Kennedy's Survey (note 3 on p. 18).

VI.

In the course of carrying out the computations for sections *II* and *III* above, several facts came to my attention, some of which I shall list below. I shall, however, refrain from giving corrections to the Halma edition of the visibility tables in the Handy Tables, for they are far too numerous. Even the ms Vat. Gr. 208 which I had occasion to consult, and which is better than Halma's text, is full of errors; this is, of course, only to be expected in tables as frequently copied as these. But with the parameters given in *III* any entry can readily be checked.

1°. In the Handy Tables the tables for Venus and Mercury have the wrong headings[1]. They should be as follows (pages refer to Halma's edition):

p. 22: Venus Σ	p. 26: Mercury Σ
p. 23: Venus Ξ.	p. 27: Mercury Γ.
p. 24: Venus Γ.	p. 28: Mercury Ξ.
p. 25: Venus Ω.	p. 29: Mercury Ω.

The tables on pp. 30 and 31 which appear under the headings of "elongation of phases" and "greatest elongation of phases" are nothing but a copy of the visibility tables in Almagest XIII, 10, for the parallel of Phoenicia. These are also found in Vat. Gr. 208 (fol. 98 v). The zīj of Ḥabash which has visibility tables for the seven climates taken from the Handy Tables also contains these Almagest tables.

2°. Venus can have so large a latitude that its projection on the ecliptic at first or last appearance is actually farther from the horizon than the sun, or so that its elongation has the opposite sign of what is normal. This abnormal situation is denoted in the manuscripts by $\overset{\varepsilon}{}$ (apparently used when Venus has greater longitude than the sun) or π (apparently used when Venus has smaller longitude than the sun) in agreement with that $\overset{\varepsilon}{}$ and π are standard abbreviations of εἰς τὰ ἑπόμενα (προηγούμενα) ζῴδια: towards the *following* (*preceding*) signs.

Halma (p. 24) cites a gloss with the words ἑπόμενα and προηγούμενα but does not seem to have realised their significance.

In the Almagest Venus at Γ in Pisces is in this abnormal situation, but there seems to be no indication of it in the texts.

3°. The following values were preferred to those given by Manitius in the table in Almagest XIII, 10, all for Mercury:

Ξ in Pisces: $11\frac{1}{2}$° (Alm. XIII, 7 B) for Manitius: 12;22
Ξ in Virgo: 18;31 (Halma p. 31) for Manitius: 18;1
Ξ in Sagit.: 22;1 (Heiberg's mss *H* and *K*) for Manitius: 20;1

The value 2;24 for β of Mercury at Ω in Pisces must be an error (see fig. 2), yet all manuscripts agree. Doubtless the computer used the β-value of the previous line.

[1] v. van der Waerden, l. c. in note 4, p. 3.

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